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Computer Simulation of Continuous Casting Processes: A Review

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Abstract: Steelmaking is the second step in producing steel from iron ore. In this stage, impurities such as sulfur, phosphorus, and excess carbon are removed from the raw iron, and alloying elements such as manganese, nickel, chromium, and vanadium are added to produce the exact steel required. Modern steelmaking processes are broken into two categories: primary and secondary steelmaking. Primary steelmaking uses mostly new iron as the feedstock, usually from a blast furnace. Secondary steelmaking uses scrap steel as the primary raw material. Gases created during the production of steel can be used as a power source. Steelmaking is presently a grounded innovation driven by plant, exploratory and computational examination. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer, and structural deformation. The important numerical modeling method of the continuous casting process has been discussed in reference in this work. With the recent advancement in metallurgical methods, the continuous casting process now becomes the main method for steel production. To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. The present work describes molten steel flow, heat transfer, solidification, electromagnetic applications, formation of the shell by solidification and coupling, etc.

Keywords: Steelmaking, Metallurgy, Computer Simulation, Continuous Casting

1. Introduction

Steelmaking is presently a grounded innovation driven by plant, exploratory and computational examination. The purpose for this is the benefits that accompany the nonstop projecting cycle which incorporates cost-saving, high efficiency, and better quality [1-6]. To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. One such kind of method has become more popular to use optimizing using numerical modeling. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer and structural deformation [7-17]. The important part and process of continuous casting have been modeled in-depth and discussed in reference [18]. It describes molten steel flow, formation of the shell by solidification. Further, the distortion of strand by thermo-mechanical forces, bulging, bending and crack prediction has been also given in detail. Till now, many powerful pre-coded solvers are available in the market. The numerical simulation of the thermo-mechanical behavior of the continuous casting process is important in terms of achieving a quality product [19-26].

This part of simulation comes with many obstacles such as dealing with the highly non-linear constitutive laws of structure, incorporation of latent heat, involvement of three different states of material: liquid, mushy and solid, temperature-dependent material properties, irregular contact between the mold surface and solidified strand, and coupling the heat transfer and structure model with proper continuum mechanism and boundary condition [27]. Reynold's Averaged Navier– Stokes (RANS) method has been widely adopted for turbulence modeling. It has been reported that the RANS model is highly accurate in predicting steady-state flow patterns [23]. The research work done in the last three decades has made continuous casting an advanced and sophisticated technology [18, 28-30]. Physical water models can simulate the molten steel flow in the mold region of the continuous casting process considering the viscosity of water equivalent to steel [31-35]. Several research works have been done on molten steel flow, heat transfer and solidification in mold [33-38]. These studies have been established and validated with industrial trials [7, 28, 39-42]. From all previous studies, it is well established that numerical models efficiently and accurately predict the fluid flow and mechanical behavior of mold and strand, respectively [31, 32]. In this present work, we have reviewed the literature to provide current information on the mathematical modeling of steelmaking tundish.

2. Computer Modeling of Mass Flow

The molten steel flow in continuous casting mold is usually assumed to have some characteristics. These flow characteristics are classified based on some assumptions such as compressible and incompressible. The molten steel flow is governed by the continuity equation and momentum equation, supplemented by heat transfer boundary conditions [43-45]. The governing equations related to mass flow and momentum transfer are as follows [46];

$$\nabla \cdot u = 0 \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + (u \cdot \nabla)u\right) = f - \nabla p + \mu \nabla^2 u \tag{2}$$

In 2005, Zhao et al. [47] studied the transient molten steel flow and superheat transport in a continuous casting mold. The equation for momentum calculation was used as follows.

$$\rho_0\left(\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial(\bar{u}_i\bar{u}_j)}{\partial x_j}\right) = -\frac{\partial \bar{p}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \bar{u}_i}{\partial x_j}\right) + \frac{\partial Q_{ij}}{\partial x_j} + \delta_{i3}\rho_0\beta(\bar{T} - T_0)g \tag{3}$$

Further momentum equation was modified by using a sub-grid momentum flux term Q_{ij}.
where,

$$Q_{ij} = \rho_0\left(\bar{u}_i\bar{u}_j - \bar{u}_i\bar{u}_j\right) \text{ Turbulence modeling} \tag{4}$$

Most of the previous work on continuous casting mold has been modeled using the RANS equation [47]. Therefore, turbulent viscosity was predicted for the Large Eddy Simulation (LES) model from the following equation.

$$\mu_T = C_v \rho_0 K_G^{1/2} \Delta \tag{5}$$

where the constant C_v is 0.05, and Δ is the grid-length scale, given by $\Delta = \left(\Delta_x\Delta_y\Delta_z\right)^{1/3}$ (Δ_x, Δ_y, and Δ_z are grid sizes in the x, y, and z directions, respectively).

To understand the complex flow profile in mold, Li and Tsukihashi [48] have developed a numerical model to investigate the vortexing flow in SEN of continuous casting of steel. To describe the behavior of vortices in the flow field, definitions of the vorticity ω_z was expressed as follows,

$$\omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial \theta}\right) \tag{6}$$

Sowa and Bokota [49] have assumed viscous incompressible and laminar flow to describes flow patterns in mold. They proposed the following equation for mass and momentum calculation.

$$\nabla . v = 0 \tag{7}$$

$$\rho \frac{dv}{dt} = \rho g - \nabla p + \mu \nabla^2 v \tag{8}$$

In a similar work in 2013, Zare et al. [50] investigated the molten steel flow filed in the mold under various conditions of submerged entry nozzle. In their work, the following momentum equation was solved;

$$\frac{\partial(\rho v_i v_j)}{\partial x_i} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\mu_{eff}\left(\frac{\partial v_i}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right)\right] + \rho g_j + F_j \tag{9}$$

In the above equation, Zare et al. (2013) [50] used the effective viscosity term in the momentum equation. The terms k and ε for turbulent viscosity were predicted from two equations of the standard k-ε model. It was expressed as follows.

$$\mu_{eff} = \mu + \mu_t \tag{10}$$

μ_t can be calculated using $k - \varepsilon$ parameters:

$$\mu_t = C_\mu \rho \frac{k^2}{\varepsilon} \tag{11}$$

The modified equation for momentum was given as follows [44];

$$\frac{\partial}{\partial t}(\rho u) + \nabla \cdot (\rho u u) = -\nabla p + \nabla \cdot [(\mu_\ell + \mu_t)\nabla u] + \rho g + \rho g \beta (T - T_0) + \frac{\left(1 - f_{\text{liq}}\right)^2}{f_{\text{liq}}^3 + 0.001} A_{\text{mush}}\left(u - u_{\text{s}}\right) + F_{\text{ave}} \tag{12}$$

In the above equation, f_{liq} the liquid fraction. The lever rule of solidification was utilized to calculate the mushy zone as follows.

$$f_{liq} = 1 - \frac{1}{1-k_0}\frac{T - T_{liq}}{T - T_{melt}} \tag{13}$$

One such popular model that works on the above-mentioned method is the k-ε model Further, more details on the mathematical modeling of multi-phase fluid flow can be read elsewhere [20, 51-53].

Comparative studies have been carried out by many researchers to investigate the influence of various turbulence models on the estimation of results [54–59]. Siddiqui et al., [60] compared different turbulence models and predicted results revealed that the RNG k-ε model has a good approximation. The equations of motion for k phase in an Euler-Euler simulation are generally given as follows:

$$\begin{gathered} \frac{\partial(\alpha_k \rho_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k u_k) = 0 \\ \frac{\partial(\alpha_k \rho_k u_k)}{\partial t} + \nabla \cdot (\alpha_k \rho_k u_k u_k) \\ = -\nabla \cdot (\alpha_k \tau_k) - \alpha_k \nabla P + \alpha_k \rho_k g' + M_{l,k} \end{gathered} \tag{14}$$

The stress terminology of k phase can be written as:

$$\tau_k = -\mu_{eff,k}\left(\nabla u_k + (\nabla u_k)^T - \frac{2}{3}\mathrm{I}(\nabla \cdot u_k)\right) \tag{15}$$

$$\mu_{eff} = \mu_{L,I} + \mu_{T,1} + \mu_{Bl,1} \tag{16}$$

Empirically the calculation of effective viscosity of gas was calculated from effective liquid velocity.

$$\mu_{ef,g} = \frac{\rho_g}{\rho_l}\mu_{eff,l} \tag{17}$$

The model proposed by Sato & Sekiguchi [22] has been used to take account of the turbulence induced by the movement of the bubbles. The expression is:

$$\mu_{BI,l} = \rho_i C_{\mu,BI}\alpha_g d_g |u_g - u_i| \tag{18}$$

$$u_k = \tilde{u}_k - u'_k \tag{19}$$

A numerical model has been developed to analyze the transient three-dimensional and three-phase flow in a bottom stirring ladle with a centered porous plug, which takes into account the steel, gas, and slag phases; it enables us to predict the fluid flow and heat transfer in the very important steel/slag region. They applied k-ε turbulence model [61];

$$\frac{\partial(\rho k)}{\partial t} + \rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu_{\text{eff}}}{\sigma_k}\cdot\frac{\partial k}{\partial x_j}\right) + G_k + G_b - \rho\varepsilon \tag{20}$$

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \rho u_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}\cdot\frac{\partial \varepsilon}{\partial x_j}\right) + \frac{(c_1 G_k \varepsilon + c_2 G_b - c_3\rho\varepsilon^2)}{k} \tag{21}$$

In the above relationship, G_k is the turbulent kinetic energy generated by mean flow velocity gradients. This can be written as follows;

$$G_k = \mu_t \frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{22}$$

$$G_k = \mu_t \frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{23}$$

Further, G_b shows the turbulent kinetic energy generated by buoyancy and it can be expressed as;

$$G_b = -g\frac{\mu_t}{\rho \mathrm{Pr}_t}\frac{\partial \rho}{\partial x_i} \tag{24}$$

The effective viscosity can be written as the addition of laminar and turbulent viscosities, as follows.

$$\mu_{\text{eff}} = \mu + \mu_t = \mu + \rho c_\mu \frac{k^2}{\varepsilon} \tag{25}$$

The values for the constants in this $k - \varepsilon$ model c_1, c_2, c_3, c_u, σ_k, and σ_ε are 1.43,1.92,0.09,1.00, and 1.30, respectively [15].

In 2014, Li et al. [62] developed a mathematical model to study the vortex formation in ladles. It is formed during liquid steel teeming from the ladle. They studied vortex formation during ladle teeming using new technology. The results obtained help to verify the validity of the numerical computations. [62]

Turbulent kinetic energy equation (k):

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left(\alpha_k \mu_{eff}\frac{\partial k}{\partial x_j}\right) + G_k + G_b - \rho\varepsilon - Y_M + S_k \tag{26}$$

Turbulent dissipation rate equation (ε)

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_j}\left(\alpha_\varepsilon \mu_{eff}\frac{\partial \varepsilon}{\partial x_j}\right) + C_{1\varepsilon}\frac{\varepsilon}{k}(G_k + C_{3\varepsilon}G_b) - C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} - R_\varepsilon + S_\varepsilon \tag{27}$$

In the past, various viscosity models have been used by the researchers to take care of the turbulence flow in the continuous casting process [15, 51, 57, 63-78].

3. Equations for Heat Transfer and Solidification

The fundamental requirement of the continuous casting process is to solidify the strand to achieve plant set quality standards [18, 30, 79-87] Generalized heat transfer equation (3-dimension) can be written in the most suitable format from the above equations in the following manner [46];

$$\rho C\left(\frac{\partial T}{\partial t} + V_x\frac{\partial T}{\partial t} + V_y\frac{\partial T}{\partial t} + V_z\frac{\partial T}{\partial t}\right) = \ddot{q} + \frac{\partial}{\partial x}\left(K_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right) \tag{28}$$

In 2005, Louhenkilpi et al. proposed a three-dimensional transient formulation for temperature distribution over the mold wall. [88];

$$\rho\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial z} = \frac{\partial}{\partial x}\left(k_{\text{eff}}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{\text{eff}}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_{\text{eff}}\frac{\partial T}{\partial z}\right) \tag{29}$$

In a similar work, Zhao et al. (2005) [47] modeled energy equation along with the Navier-Stokes equation.

$$\frac{\partial \bar{T}}{\partial t} + \frac{\partial(\bar{u}_i\bar{T})}{\partial x_i} = \frac{k}{\rho_0 C_P}\frac{\partial}{\partial x_i}\left(\frac{\partial \bar{T}}{\partial x_i}\right) + \frac{\partial Q_{Ti}}{\partial x_i} \tag{30}$$

$$Q_{Ti} = \bar{T}\bar{u}_i - \overline{Tu_i} \tag{31}$$

$$\frac{\partial Q_{Ti}}{\partial x_i} = \frac{\mu_T}{\mathrm{Pr}_T}\frac{\partial}{\partial x_i}\frac{\partial \bar{T}}{\partial x_i} \tag{32}$$

In 2011, Sowa and Bokota [49] proposed a heat flow model based on the Fourier-Kirchhoff system of equations.

$$\rho c\left(\frac{\partial T(\mathbf{x},t)}{\partial t} + \nabla T\cdot\mathbf{v}\right) = \nabla\cdot(\lambda\nabla T) + \dot{Q} \tag{33}$$

Sowa and Bakota et al. [49] modified the above equation which includes effective specific heat (Ceff) term which is a function of the temperature of the material.

$$\nabla.(\lambda\Delta T) - C_{ef}\frac{\partial T}{\partial t} - C_{ef}\nabla T.V = 0 \tag{34}$$

$$C_{ef}(T) = \rho_{LS}c_{LS} + \rho_S L/(T_L - T_S) \tag{35}$$

In 2011, Hadata et al., [37] proposed a steady Fourier-Kirchhoff model for heat flow with some assumptions.

$$q_v = Q_s\frac{dV_s}{d\tau} \tag{36}$$

In a study in 1993 S. E. Chidiac et. at., [64] used enthalpy approach for heat transfer in multi-dimensional problem with following equation.

$$\rho \frac{\partial H}{\partial t} = \nabla \cdot (K\nabla T) + Q \quad (37)$$

where ρ indicates density, H indicates enthalpy, K indicates Thermal conductivity, Q indicates heat generation rate for unit volume, T indicates temperature and t time. Enthalpy is nothing but the summation of sensible & latent heat and can be expressed as:

$$H = \int_{T_r}^{T} cdT + f(T).L \quad (38)$$

where c, *f(T)* and *L* are specific heat liquid fraction and latent heat. For phase change study two methods are clubbed together with the above-stated formulation for accuracy and efficiency. Dirichlet & Cauchy boundary conditions are used to solve above equations. The study carried in 2003, B. wiwanapataphee et. al., [63] for simulating phase change cause of heat transfer single domain enthalpy method is adopted. Where enthalpy is the summation of latent heat (H) & sensible heat (h).

$$H=h+\Delta H \quad (39)$$

Latent heat h can be given by

$$H=f(T)L, \quad (40)$$

Where L denoted Latent Heat of Steel *L* and *f*(*T*) indicates localized liquid fraction where value one represents complete Liquids state and zero represents the complete solid-state. The liquid fraction is nonlinear for simplification of the model it is assumed linear.

$$f(T) = \begin{cases} 0, & T \leqslant T_s \\ \frac{T-T_S}{T_L-T_S}, & T_s < T < T_L, \\ 1, & T \geqslant T_L, \text{ (XX)} \end{cases} \quad (41)$$

Where in T_L indicates melting temperature and T_S Solidification temperature.

For region where phase change occurs conservation of energy principle. Combining this equation with enthalpy gives,

$$\rho c \left(\frac{\partial T}{\partial t} + u_j T_,\right) = \left(k_0 T_j\right)_j - S_T \quad (42)$$

Ivanova (2013) [89] formulated extensive mathematical modeling on predicting phase-dependent boundary conditions.

$$\frac{\partial T}{\partial r} + v(t) \cdot \frac{\partial T}{\partial z} = \frac{1}{c(T)\rho(T)} \times \\ \times \left\{\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial \partial_z}\left[\lambda(T)\frac{\partial T}{\partial_z}\right]\right\} \quad (43)$$

The position of the unknown phase boundary is specified by the equality condition of the temperatures and the Stefan condition for the two-dimensional case:

$$T = T(\tau, x, z)|_{x=\xi\_(\tau,z)=} T(\tau, x, z)|_{x=\xi_+(\tau,z)=T_{cr}} \quad (44)$$

$$\lambda(T)\frac{\partial T}{\partial \bar{n}}|\xi_+ - \lambda(T)\frac{\partial T}{\partial \bar{n}}|\xi_- = \mu\rho\,(T_{kp})\left(\frac{d\xi}{d\tau} + v(\tau)\frac{d\xi}{dz}\right) \quad (45)$$

where ξ is the phase boundary $x = \xi(\tau, z), \bar{n}$ is a normal to the phase boundary, $\frac{\partial T}{\partial \pi}\Big|_{\xi+/-}$ is the left-right limit of the temperature derivative in the normal direction. μ is the latent the heat of crystallization. T_{cr} is the crystallization temperature (the average temperature from the liquidus-solidus interval).

In 2014, Zhang et al [90] investigated a steady-state two-dimensional numerical model based on the assumption of heat transfer.

$$\rho = (1 - f_S)\rho_L + f_S(f_\delta \rho_\delta + f_\gamma \rho_\gamma) \quad (46)$$

$$\lambda = (1 - f_S)\lambda_L + f_S(f_\delta \lambda_\delta + f_\gamma \lambda_\gamma) \quad (47)$$

$$c_{\text{eff}} = f_S \cdot c_S + (1 - f_S) \cdot c_L - L\frac{\partial f_S}{\partial T} \quad (48)$$

In a similar work, Maurya and Jha (2014) [91] investigated the effect of casting speed in the continuous casting process.

$$\rho \frac{\partial H}{\partial t} + \rho\nabla \cdot (uH) = \nabla(k_{eff}\nabla T) + Q_\ell \quad (49)$$

Where ρ is density, H is enthalpy, ΔH is sensible heat, QL is source term. QL can be expressed as a single solidification model and given as;

$$Q_L = \rho L\frac{\partial f_s}{\partial t} + \rho L\bar{u}_{pull} \cdot \nabla f_s \quad (50)$$

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad (51)$$

Naiver-Stokes equation for transient momentum conservation is given by

$$\frac{\partial}{\partial t}(\rho u) + \rho\nabla(uu) = -\nabla P + \nabla\{\mu_{eff}(\nabla \cdot u)\} + \rho + S \quad (52)$$

where,

$$\mu_{eff} = \mu_l + \mu_t$$

Maurya and Jha (2014) [91] and Hitanen et al. (2017) [92] used the enthalpy-porosity technique for solidification.

$$S = \frac{(1-\beta)^2}{(\beta^3-\xi)} A_{\text{mush}}\left(\bar{u} - \bar{u}_{\text{pull}}\right) \quad (53)$$

where, liquid fraction is expressed as β, ξ = 0.001, mushy zone constant is given as A_{mush}.

Pilvi et. at., (2017) [65] Used turbulent flow modelling at inlet in which they considered hydraulic diameter at inlet.

$$\lambda_e = \lambda_L L(1 + 6(1 - f_s))^2 \quad (54)$$

In 2016, Hibbeler et al. [93] proposed an innovative reduced-order model (ROM) for heat transfer from mold in the continuous casting of steel.

$$0 = \frac{\partial^2\theta_{\text{mould}}}{\partial x^{*2}} + \left(\frac{d_{\text{mould}}}{w_{\text{mould}}}\right)^2 \frac{\partial^2\theta_{\text{mould}}}{\partial y^{*2}} + \left(\frac{d_{\text{mould}}}{\ell_{\text{mould}}}\right)^2 \frac{\partial^2\theta_{\text{mould}}}{\partial z^{*2}} \quad (55)$$

Vnnyscy and Saleem (2017) [94] formulated a mathematical formulation for explicitly calculating the geometrical range of the mushy zone.

$$\rho c_{\text{p}} V_{\text{cast}} \frac{\partial T}{\partial z} = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - \rho V_{\text{cast}} \Delta H_{\text{f}} \frac{\partial \chi}{\partial z} \quad (56)$$

where

$$k = \chi k_1 + (1-\chi)k_s$$

$$c_{\text{p}} = \chi c_{\text{pl}} + (1-\chi)c_{\text{ps}}$$

A decoupled three-dimensional mathematic model of fluid flow and heat transfer in continuous casting billet mould was developed by An et al., (2018) [95].

$$\frac{\partial}{\partial t}(\rho H) + \frac{\partial}{\partial x_j}(\rho\mu_j H) = \frac{\partial}{\partial x_j}\left[\left(\lambda + C_p\frac{\mu_t}{\sigma_t}\right)\frac{\partial H}{\partial x_j}\right] \quad (57)$$

Ole Richter et al. (2017) [96] studied the development of free surface flow for the liquid and/or solid phase change. They considered enthalpy-porosity and volume-of-fluid (VOF) method.

$$\alpha_1 = \begin{cases} 0 & = \text{gas} \\ 0 < \alpha_1 < 1 & = \text{cell contains the interface} \\ 1 & = \text{solid or liquid PCM} \end{cases} \quad (58)$$

The molten steel fraction was completely dependent on the thermal condition (T) of liquid metal. TS and TL indicates same respectively. This can be expressed as follows [96];

$$\gamma_{1,l} = \begin{cases} 0 & \text{if} \quad T < T_S \\ \frac{T-T_S}{T_L-T_S} & \text{if} \quad Ts \le T \le T_L. \\ 1 & \text{if} \quad T > T_L \end{cases} \quad (59)$$

Where one indicates complete liquid state and zero indicates complete solid state. In between values of solid fraction indicates mushy zone.

In the given formulation the density ρ, the heat capacity cp, the heat conduction λ and the viscosity μ can be expressed as follows;

$$\rho = \alpha_1(\gamma_{1,l}\rho_{1,l} + \gamma_{1,s}\rho_{1,s}) + \alpha_2\rho_2 \quad (60)$$

$$c_p = \alpha_1(\gamma_{1,l}c_{p1,l} + \gamma_{1,s}c_{p1,s}) + \alpha_2 c_{p2} \quad (61)$$

$$\lambda = \alpha_1(\gamma_{1,l}\lambda_{1,l} + \gamma_{1,s}\lambda_{1,s}) + \alpha_2\lambda_2 \quad (62)$$

$$\mu = \alpha_1\mu_{1,l} + \alpha_2\mu_2 \quad (63)$$

In above equations, the subscripts l, 1, s and 2 illustatre the property of the bulk liquid, solid and gas phase, respectively. In order to consider natural convection in proposed numerical formulation, the Boussinesq approach was used. Further, the buoyancy modified density ρb can be defined as;

$$\rho_b = \alpha_1(\gamma_{1,l}\rho_{1,l}(1-\beta(T-T_L)) + \gamma_{1,s}\rho_{1,s}) + \alpha_2\rho_2 \quad (64)$$

4. Thermo-mechanical Deformation

The behavior of metal especially steel at high temperature becomes sensitive to strain rate and temperature. Therefore, process design of hot metal working of steel is significantly affected by non-linear behavior of steel. Structural distortion arises in mold and strand due to thermal distribution, which causes thermal stress, cracks and ultimately affects quality strand [35, 97]. Many research has been done on mould thermal distortion in mould and strand [7, 8, 98-100]. In 2006, To measure surface temperature and shell thickness, finite point method was used by Alizadeh et al. [2]. It has been also reported heat transfer rate is affected by mold distortion [98, 101]. Many research has been done on mould thermal distortion in mould and strand [98, 99]. Generally, the heat transfer equation is solved with interfacial heat flux data and it is quantified from plant data. Subsequently, equations related to thermo-mechanical distortion in mold and strand is calculated.

$$d\varepsilon_{ij} = d\varepsilon_{ij}{}^{e} + d\varepsilon_{ij}{}^{p} \quad (65)$$

where $d\varepsilon_{ij}{}^{e}$ and $d\varepsilon_{ij}{}^{P}$ are the incremental elastic and plastic components of the total strain vector $d\varepsilon_{ij}$

In this work they proposed incremental stresses and strains during plastic flow;

$$d\varepsilon_{ij}{}^{P} = d\lambda \frac{\partial Y}{\partial \sigma'_{ij}} \quad (66)$$

where dλ is a scalar multiplying factor, dY is derivative of yield stress and σ_{ij} is the deviatoric stress vector.

In 2000, Lee et al. [102] proposed a modified model of thermo-mechanical deformation in strand. They developed a mathematical model for the coupled analysis. The coupled analysis consisted of various mathematical models. The coupled model considered molten steel flow characteristics in mould. Further, it coupled the and heat transfer, thermo-mechanical deformation behavior of a solidifying strand in the continuous casting process. Moreover, Von-mises yield function and associated flow were assumed for increment of stress. The stress in thermo-elasto-plastic material can related as;

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{P} - \varepsilon_{kl}^{Th}) \quad (67)$$

where $C_{ijkl}, \varepsilon_{kl}, \varepsilon_{kl}^{P}$, and ε_{kl}^{Th} are the elastic constitutive matrix, total infinitesimal strain, plastic strain, and thermal strain, respectively.

In a similar work, Ha et al., (2000) [79] carried a mathematical modeling for heat transfer study in secondary cooling zone of continuous casting strand. It was reported that creep was dominant factor in bulging defect. The elastic-plastic creep model for the strand is given by:

$$\dot{\varepsilon} = \alpha\sigma^m \quad (68)$$

where $\sigma(\text{kg/cm}^2)$ and $\dot{\varepsilon}(\text{I/s})$ denote the equivalent stress and the creep strain rate, respectively, and m is a constant of 3.15. Also

$$\alpha = 0.0806\exp\left\{-\frac{28392}{T+273}\right\} \tag{69}$$

In 2004, Bellet et al. [8] introduced a global non-steady state (GNS) method for liquid-solid constitutive model which considered mushy zone during solidification. They reported the following relationship for total strain calculation in liquid and mushy zone;

$$\dot{E} = \dot{\varepsilon}^{\mathrm{vp}} + \dot{\varepsilon}^{\mathrm{th}} \tag{70}$$

where Evp is a strain in visco-plastic condition and Eth strain due to thermal expansion. In addition to this, a thermo-elastic–viscoplastic model was used to represent the behavior in the solid state. It was described by the following equations [8];

$$\dot{\varepsilon} = \dot{\varepsilon}^{\mathrm{el}} + \dot{\varepsilon}^{\mathrm{vp}} + \dot{\varepsilon}^{\mathrm{th}} \tag{71}$$

In a similar work, Liu and Zhu (2006) [103] assumed mould copper plate should exhibit thermoelastic behavior and thermoelastic-plastic behaviour for strand. The isotropic linear elastic stress–strain relation was expressed by the constitutive equation as follows:

$$\sigma_{ij} = 2G\varepsilon_{ij} + [\lambda\varepsilon_{kk} - (3\lambda + 2G)\alpha\Delta T]\delta_{ij} \tag{72}$$

It was reported that the total strain can be expressed as the sum of an elastic strain, a thermal strain, and a plastic strain as follows;

$$\varepsilon_{ij} = \varepsilon_{ij}^{\mathrm{e}} + \varepsilon_{ij}^{\mathrm{p}} + \varepsilon_{ij}^{\mathrm{T}} \tag{73}$$

where, temperature change ΔT may induce a thermal strain of a magnitude

$$\varepsilon_{ij}^{\mathrm{T}} = \alpha\Delta T\delta_{ij} \tag{74}$$

In a recent work, Li et al. (2017) [104] reported that in the mushy zone, the stress in solid steel is supposed to increase linearly with the rise in solid fraction between zero strength temperature (T_{ZST}) at f_s*=0.75 and* zero Xdeformation temperature ($T_{ZDT)}$ and it can be given as follows;

$$\sigma_{\mathrm{s}} * \frac{(f_{\mathrm{s}} - f_{\mathrm{ZST}})}{1 - f_{\mathrm{ZST}}} \tag{75}$$

where f_s is solid fraction stress, f_{zst} is stress at zero strength temperature.

Several authors have predicted the probability of crack formation in solid strand by crack susceptibility coefficient SC as follows [102, 105, 106];

$$\begin{aligned} S_C &= \frac{Y_M}{Y_C} \text{ for } {}^T f_S \le f_S < 1 \\ &= 0 \text{ for } 0 \le f_S < T_S \\ &= 0 \text{ for } Y_M \le 0 \end{aligned} \tag{76}$$

where ε_e elastics strain, ε_p Plastic Strain, ε_T Thermal strain. Thermal strain is given by

$$\Delta\{\varepsilon\}_{\mathrm{T}} = \left(\{\alpha\} + d[\frac{D]_{\mathrm{c}}^{-1}\{\sigma\}}{dT}\right) dT \tag{77}$$

where α indicates coefficient of thermal expansion. Further in elastic region stress given by

$$\Delta\{\sigma\} = [D]_{\mathrm{e}}(\Delta\{\varepsilon\} - \Delta\{\varepsilon\}_{\mathrm{T}}) \tag{78}$$

where D_e Indicates Elastic-Plastic matrix. σ Indicates stress. Further in the plastic region the stress is given by

$$\Delta\{\sigma\} = [D]_{\mathrm{ep}}\,(\Delta\{\varepsilon\} - \Delta\{\varepsilon\}_{\mathrm{T}}) + \Delta\{\sigma\}_{\mathrm{T}} \tag{79}$$

Hence the thermal stress is

$$\Delta\{\sigma\}_{\mathrm{T}} = \frac{[D]_C((\partial\bar{\sigma}\,\partial H)/(\partial\{\sigma\}\,\partial T))dT}{H' + \{\sigma\bar{\sigma}/\partial\{\sigma\}\}^{\mathrm{T}}[D_{ec}(\partial\bar{\sigma}/\partial\{\sigma\})} \tag{80}$$

where $\bar{\sigma}$ indicates equivalent stress at node.

They noted that near the meniscus liquid fraction is more compared to bottom slab. It shows that solidification is start early at bottom side. Because of uneven temperature in slab leads to thermal strain which creates thermal stress.

Hadata et al., [11] studied surface crack defect evaluation four criteria used namely plastic work criteria, Rice and Tracy Criteria, modified Rice and Tracy criteria and Latham criteria. Plastic work criteria can be given by following equation

$$C_{EP} = \int_0^1 \bar{\varepsilon}\bar{\sigma}dt \text{ for } \sigma_m > 0 \tag{81}$$

where ε indicates strain rate, σ indicates stress. This criteria based on assumption that crack will get generated if strain energy is more than critical value C_{EP}. Plastic strain is evaluated only in region where mean stress is positive.

Following is the criteria given by Rice & Tracy

$$C_{RT} = \bar{\varepsilon}\exp\left(-\frac{3}{2}\frac{6m}{\bar{\sigma}}\right) \tag{82}$$

where σ_m is mean stress & ε indicates strain. This criteria assumes that crack will appear if strain increases beyond C_{RT}.

Following is the criteria given by modified which uses only positive values of strain for calculation of critical parameter C_{RM}. The

$$C_{RM} = \sum\Delta\bar{\varepsilon}\exp\left(-\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\right) \text{ for } \sigma_m > 0 \tag{83}$$

For Latham Criteria equation is as follows

$$C_{LO} = \int_0^t \sigma_{\max}\dot{\varepsilon}\,\mathrm{dt} \text{ for } \sigma_m > 0 \tag{84}$$

5. Conclusions

Many literatures have reported about strand bulging between rolls which have caused transverse cracks, radial streaks and centerline macrosegregation [7, 8, 100]. Risso et al. [107] evaluated the thermal stress and strain in the solidifying shell of the strand by using the analytical method. Recently, Chen et al. (2019) [108] investigated the mold level fluctuations. These fluctuations are caused by transient bulging of the solidifying shell. Consequently, transient bulging phenomenon affects the quality of the steel. They developed a 1D and 2D model for strand simulation. They reported that mold level fluctuations are highly caused by dynamic bulging. Several constitutive models have been adopted for simulating the solidification stresses using the simple elastic-plastic models [109, 110]. Researchers added a separate creep model for transient modeling [111]. The integration of these transient constitutive laws and further, mathematical modeling is a challenging task. From all the above discussion it is observed that the temperature and stress-strain distribution in the strand region of the continuous casting process plays an important role in defining the quality of the final solidified product [27, 112-114]. In 2006, Liu and Zhu [103] developed a three-dimensional finite-element heat-transfer and thermal stress models to study the thermo-mechanical distortion on the slab during operation. They reported that operating parameters i.e., casting affected the strand distortion in copper walls of the mould. Pascon and coworkers (2006) [115] studied the generation of transverse crack during bending and straightening of strands. The numerical model was applied and validated with industrial data. The transverse cracks were found at the upper face of the strand. A numerical model was presented by Fachinotti et al. (2006) [100] to study the macro-segregation defects in strand caused by thermal stress. They made a hypothesis about the transient effect of alternate rolling and bulging. To measure surface temperature and shell thickness, finite point method was used by Alizadeh et al. [2]. They compared FPM results with FVM results. It was concluded that heat transfer, surface temperature, and shell thickness can be successfully modeled by FPM method.

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